

Excerpt from an early draft of the book "Discovering the Art of Mathematical Games and Puzzles," in the series of inquiry-based learning guides entitled "Discovering the Art of Mathematics" by Julian Fleron, Philip Hotchkiss, Volker Ecke, and Christine von Renesse, supported by a CCLI grant from the National Science Foundation. For further details, see <http://artofmathematics.wsc.ma.edu/> or contact Volker Ecke at [vecke@wsc.ma.edu](mailto:vecke@wsc.ma.edu).

## 1. CONNECTION GAMES: HEX, CONHEX, ...

**1.1. Rules of Hex and Playing Hex.** According to Wikipedia, Hex is a board game played on a hexagonal grid, theoretically of any size and several possible shapes, but traditionally as an 11x11 rhombus. Other popular dimensions are 13x13 and 19x19 as a result of the game's relationship to the older game of Go. According to the book *A Beautiful Mind*, John Nash (one of the game's inventors) advocated 14x14 as the optimal size.

Each player has an allocated color, Red and Blue being conventional. In this book, we will use Black and Gray instead. Players take turns placing a stone of their color on a single cell within the overall playing board. The goal is to form a connected path of your stones linking the opposing sides of the board marked by your colors, before your opponent connects his or her sides in a similar fashion. The first player to complete his or her connection wins the game. The four corner hexagons each belong to two sides. As you place a stone, you don't have to place it adjacent to another of your color. Any open hexagon on the board can be chosen.

In class, we have opponents to play against. If you want to practice at home or explore strategies on your own, you can find Hex applets online<sup>1</sup> or download an application. For some of these, you can play against a computer or against somebody else on the net.

### **Explorations:**

- (1) Play the game a certain number of times. Write down your strategies.
- (2) Try playing against different people.
- (3) If you have a partner, both of you should speak out loud your reasons for every single move. (While this gives away what you're doing, you will both also learn a lot.)
- (4) Where do you think is the best place for the first stone? Explain.

**1.2. Examples.** Consider the board in Figure (1). Notice that the gray stone has a white circle inside it, indicating that this was the more recent move on the board. Black's turn to play.

- (5) Playing for black, where would you place the next stone?
- (6) Playing for gray, how would you counter these moves?
- (7) Continue the game to see whether your choices were good choices.
- (8) Working with a partner, take turns to explore different strategies.

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<sup>1</sup> The most popular sites are boardspace for real-time play, <http://www.ludoteka.com/> for real-time play, <http://games.wtanaka.com/hex/> for real-time or turn-based play, igGameCenter (<http://www.iggamecenter.com/>) for real-time play, with time settings, and ranking, and <http://havannah.vying.org/> for real-time play (Hex/Havannah). Strong game applications include *Six* by Gábor Melis and *Hexy*.

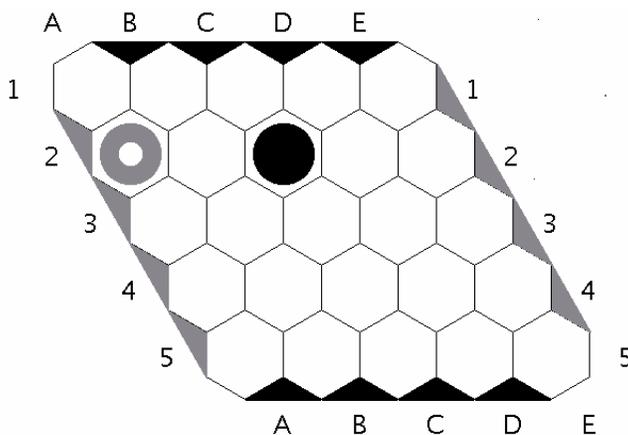


FIGURE 1. Black's turn to play. Which is the best move?

(9) On your own, write down your detailed observations and strategies. Now consider the board in Figure (2). Black's turn to play.

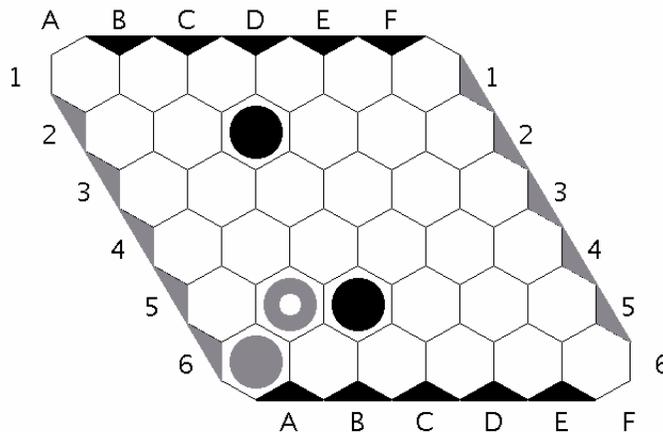


FIGURE 2. Black's turn to play. Which is the best move?

- (10) Playing for black, where would you place the next stone?
- (11) Playing for gray, how would you counter these moves?
- (12) Continue the game to see whether your choices were good choices.
- (13) Working with a partner, take turns to explore different strategies.
- (14) On your own, write down your detailed observations and strategies.

**1.3. History of Hex.** According to Wikipedia, the game was invented by the Danish mathematician Piet Hein, who introduced the game in 1942 at the Niels Bohr Institute, and also independently invented by the mathematician John Nash in 1947 at Princeton University. It became known in Denmark under the name Polygon (though Hein called it CON-TAC-TIX); Nash's fellow players at first called

the game Nash. According to Martin Gardner, some of the Princeton University students also referred to the game as John (according to some sources this was because they played the game using the mosaic of the bathroom floor.). However, according to Sylvia Nasar's biography of John Forbes Nash *A Beautiful Mind*, the game was referred to as "Nash" or "John" after its apparent creator. John Nash was said to have thought of this game, independent of Hein's, during his graduate years at Princeton. In 1952 Parker Brothers marketed a version. They called their version "Hex" and the name stuck. Hex is an abstract strategy game that belongs to the general category of "connection" games. Other connection games include Omni, Y and Havannah. All of these games are related to the ancient Asian game of Go; Nash's version of Hex, in particular, was done as a response to Go.

**Exploration:** Research further information about the people mentioned above or about some of the other "connection" games that are mentioned and write a short paper about your findings.

1.4. **Edge Templates.** In order to better understand some strategies underlying Hex, let us consider so-called *Edge Templates*; see Figure (3).

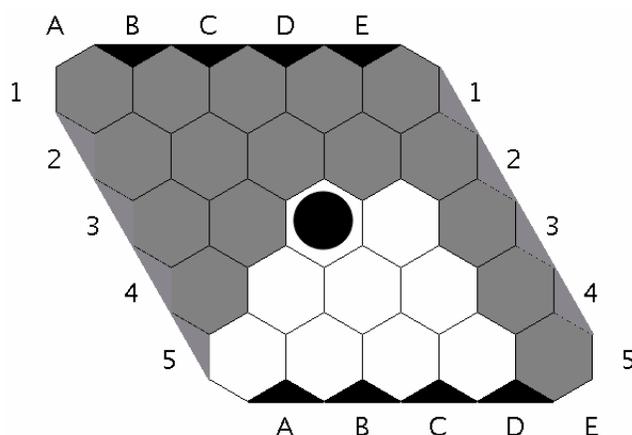


FIGURE 3. How to block Black from reaching the edge?

**Rules:** Stones can now be placed only in the unshaded hexagons. Black and Gray have different goals: Black's goal is to reach the lower black edge. Gray is trying to block Black from doing so. Gray to go first.

**Exploration:**

- (15) With a partner, taking turns, explore the edge template.
- (16) What do you notice?
- (17) Is there a pattern in who is able to win?
- (18) If you notice that one of the players always wins, how can you be sure that this will also happen when playing against other people? Explain.
- (19) Play against your teacher to check your strategy.

Next consider all the above questions for the Edge Template in Figure (4). If you feel you have a strategy, play against your teacher.

Finally, consider all the above questions for the Edge Template in Figure (5). If you feel you have a strategy, play against your teacher.

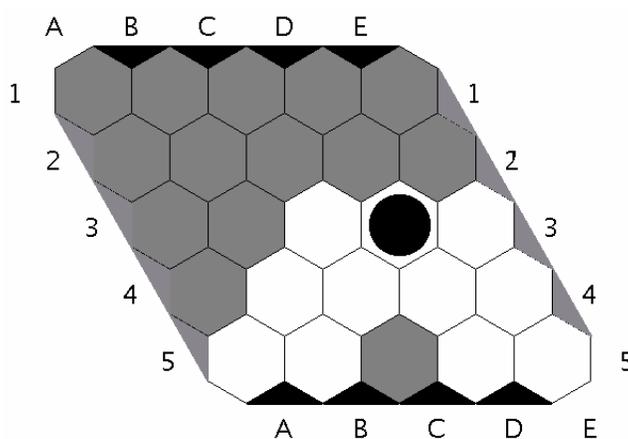


FIGURE 4. How to block Black from reaching the edge?

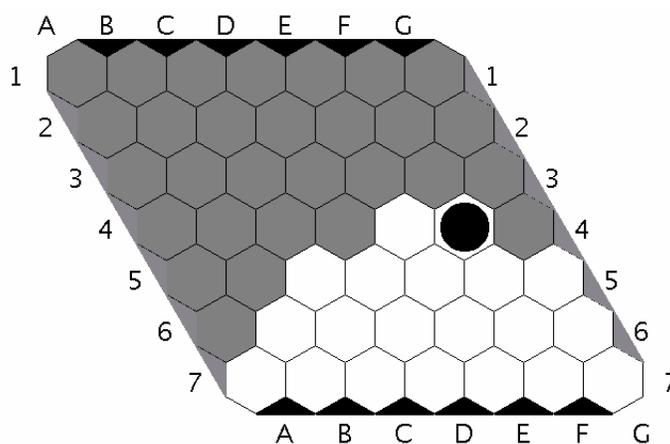


FIGURE 5. How to block Black from reaching the edge?

### 1.5. Good moves.

- (20) Could you place additional edge templates into the board shown in Figure (3)?
- (21) What do you now think about the best place to start the game? Explain.
- (22) How did your experience with the edge templates influence your thinking on starting the game?
- (23) "Skip one:" Here is possible strategy: Suppose your opponent has the strategy of "skipping one." How would you counter that strategy.
- (24) Can this person be blocked? If not, explain why. If so, list the precise counter-moves.